Kaon Fragmentation Function from NJL-Jet

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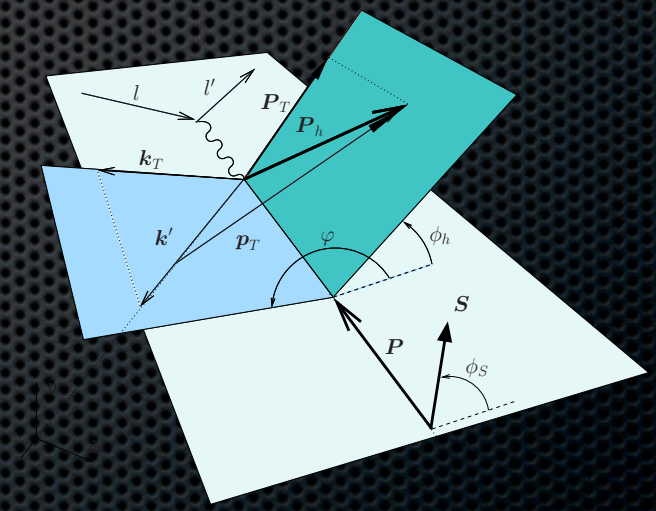
Outlook

- Motivation
- Strange NJL-jet: Distribution & Fragmentation Functions
- Monte-Carlo simulations and inclusion of $p_\perp$

Hadron Structure with SIDIS

A. Kotzinian, Nucl. Phys. B441, 234 (1995).

- Semi-inclusive deep inelastic scattering (SIDIS): $e\,N \rightarrow e\,h\,X$
- Cross-section factorizes into parton distribution and fragmentation functions.



Access to hadron structure:

- Ex., unpolarized cross section is ~

$$\sum_q e_q^2 \int d^2\mathbf{k}_\perp f_1^q(x, k_\perp)\pi y^2 \frac{\hat{s}^2 + \hat{u}^2}{Q^4} D_q^h(z, p_\perp)$$

- NJL provides a sound framework for calculating both!

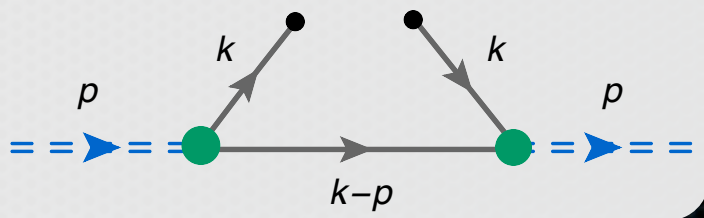
Distribution and Fragmentation

Ito et al. Phys.Rev.D80:074008,2009

Probability of finding quark q(x) in hadron h:

$$f_q^h(x) = p_- \int d^2k_T \sum_\alpha \frac{\langle p|b_\alpha^\dagger(p) b_\alpha(p)|p\rangle}{\langle p|p\rangle}$$

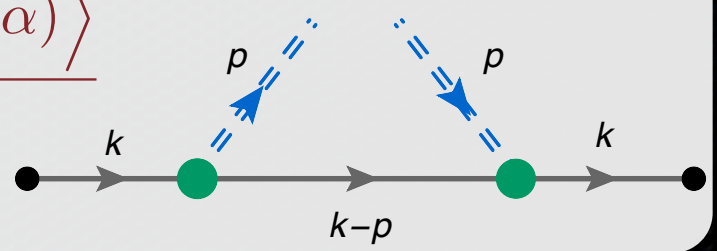
$$x = k_-/p_-$$



Probability of finding hadrons h(z) in cloud of q:

$$D_q^h(z) = \frac{1}{6} dp_- \int d^2p_\perp \sum_\alpha \frac{\left\langle k(\alpha)|a_h^\dagger(p) a_h(p)|k(\alpha)\right\rangle}{\langle k(\alpha)|k(\alpha)\rangle}$$

$$z = p_-/k_-$$

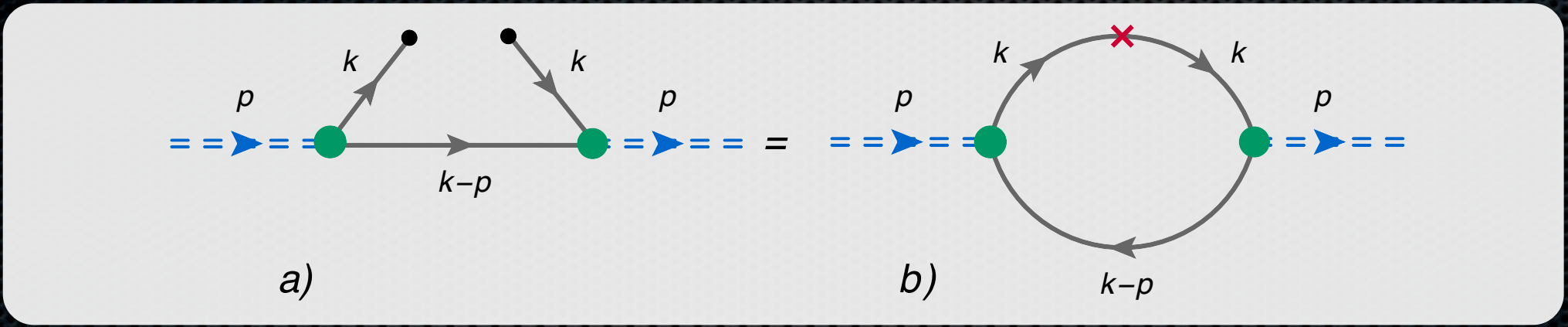


Motivation

- Effective quark model descriptions of fragmentation functions usually employ “elementary” one-step process.
- The resulting fragmentation functions are too small compared to data (e.g. M. Hirai et al: PRD 75 (2007) 094009.).
- Multiple hadron emission is mimicked by introducing “normalization factors”.

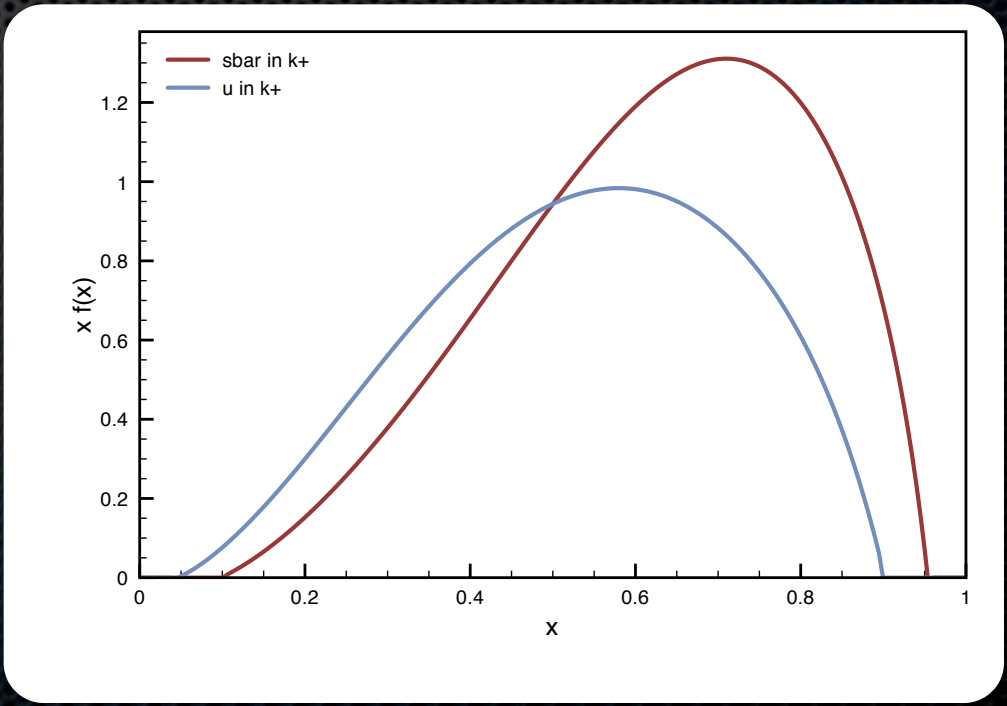
K Distribution Function in NJL

Bentz et al.: Nucl.Phys.A651:143-173,1999.



- Regularization:

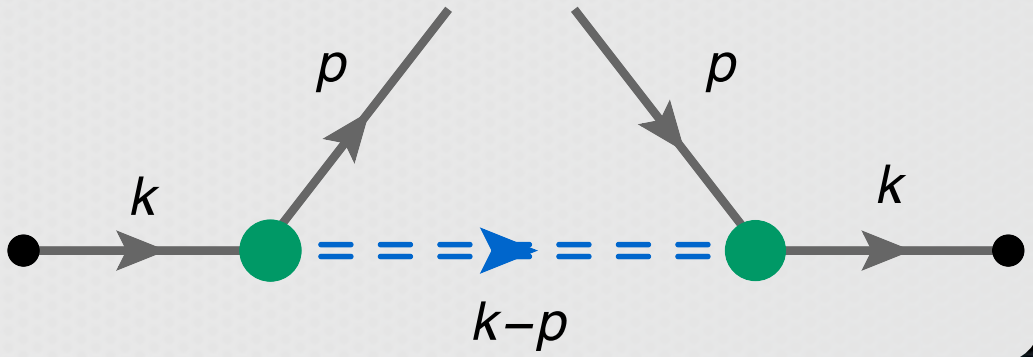
  Lepage-Brodsky (LB)
  Invariant Mass Cutoff



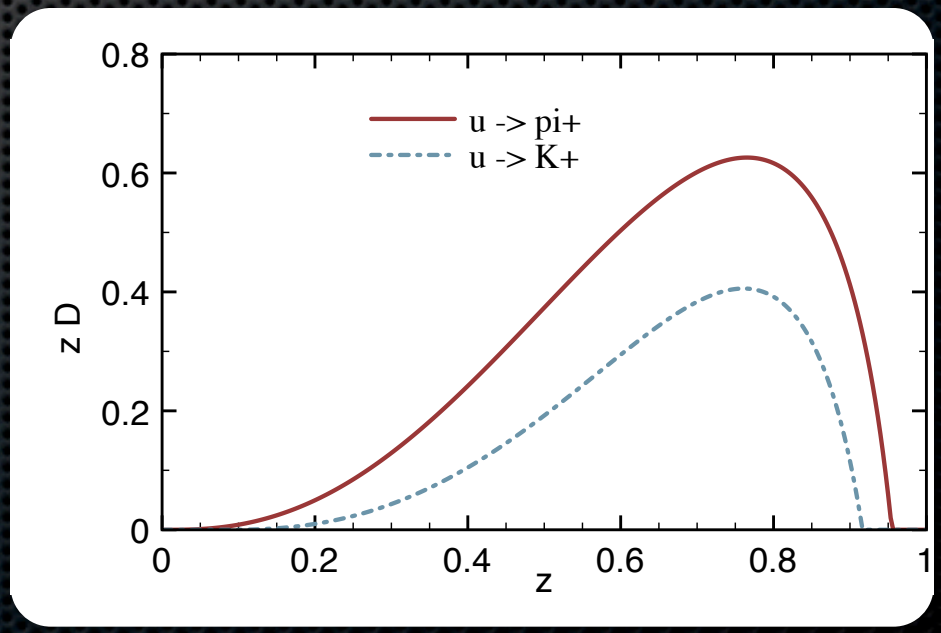
Splitting Functions

- One-quark truncation of the wavefunction:

$$d_q^m(z) : q \to Qm$$
$$m = q\bar{Q}$$



$$u \to d\pi^+$$
$$u \to sk^+$$

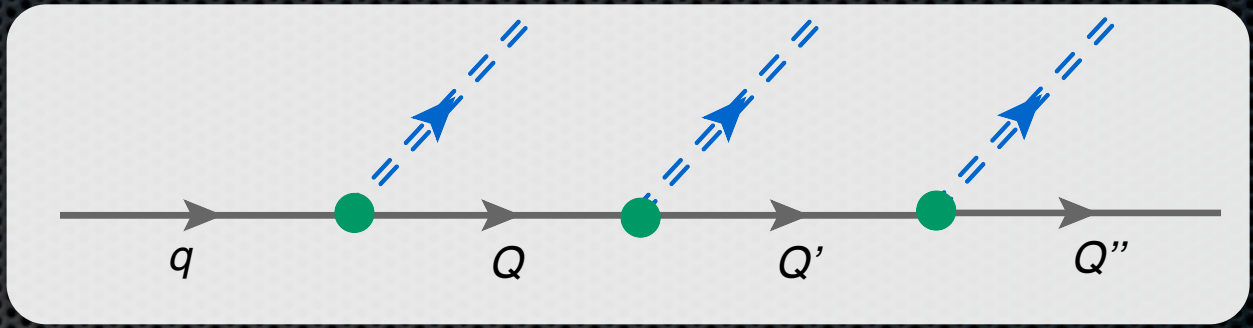


NJL-jet Model for Fragmentation Function

Field, Feynman.Nucl.Phys.B136:1,1978.

- Chain Decay:

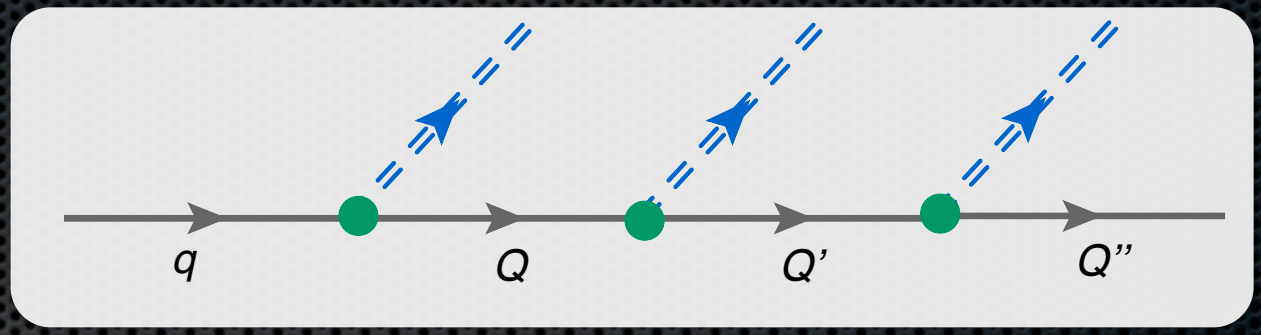
  No re-absorption



$$D_q^m(z) = \hat{d}_q^m(z) + \int_z^1 \frac{dy}{y} \hat{d}_q^Q(\frac{z}{y}) \cdot D_Q^m(y)$$

$$\hat{d}_q^m(z) = \hat{d}_q^{Q'}(1-z)|_{m=\bar{Q}'q}$$

Probabilistic Interpretation of Integral Equation



The probability of finding mesons m with mom. fraction z in a jet of quark q

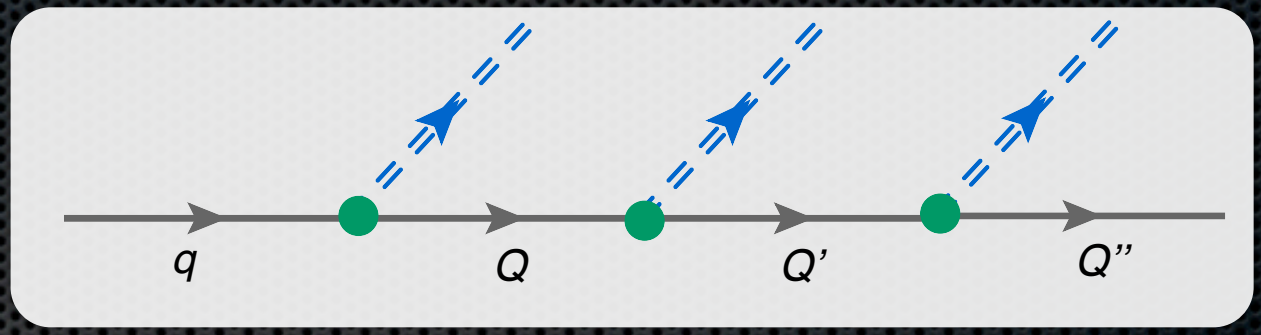
$$D_q^m(z)dz = \hat{d}_q^m(z)dz + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^m(\frac{z}{y})\frac{dz}{y}$$

Probability of emittng the meson at link 1

Probability of Momentum fraction y is transferred to jet at step 1

The probability scales with mom. fraction

Probabilistic Interpretation of Integral Equation



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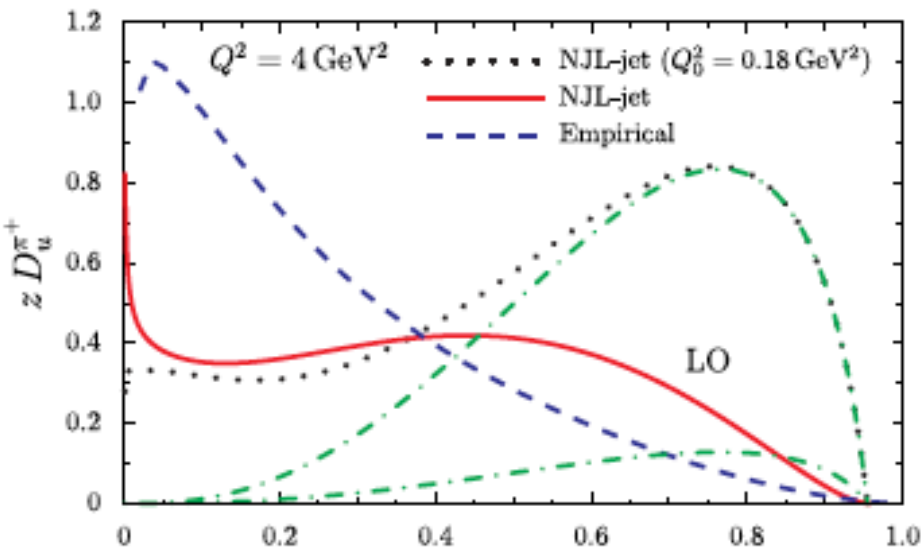
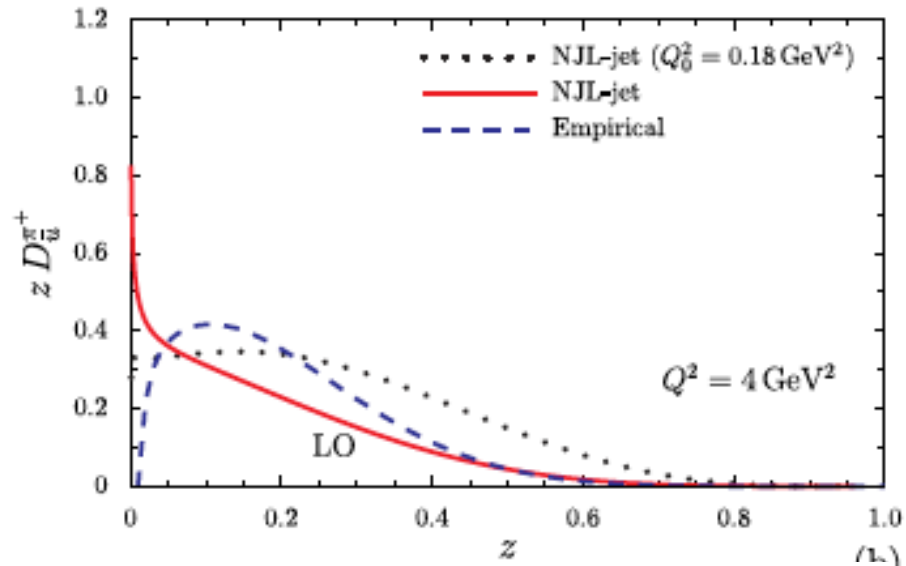
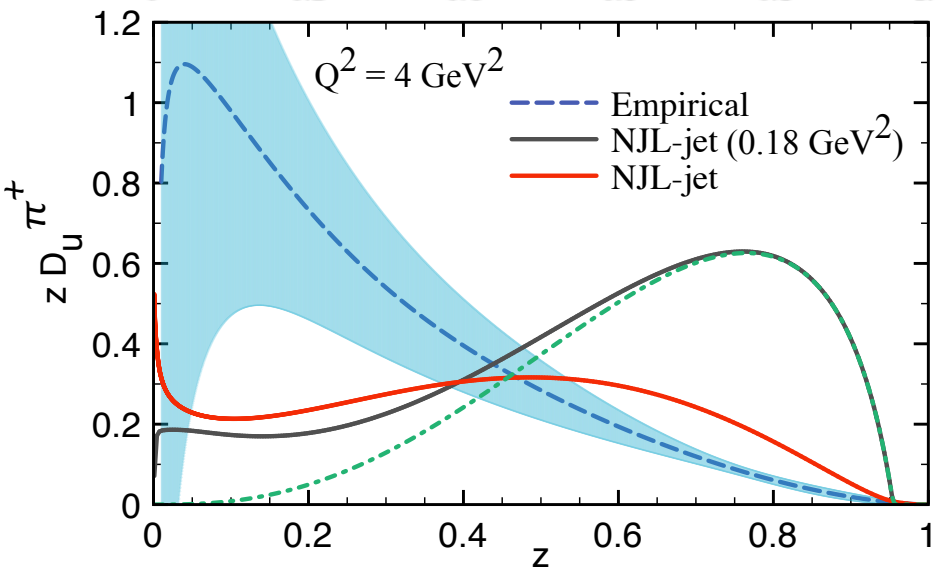
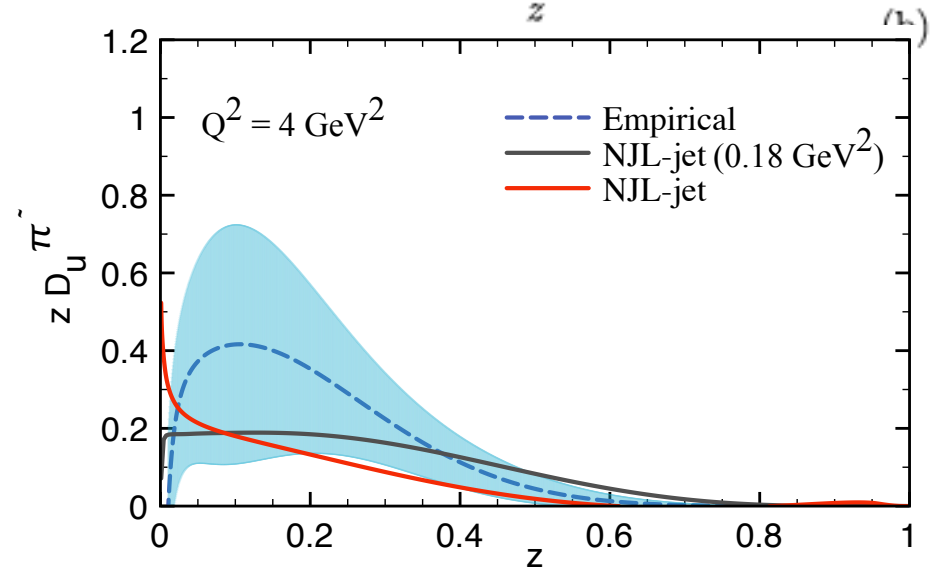
Strangeness Effect in Pion

Ito et al. Phys.Rev.D80:074008,2009

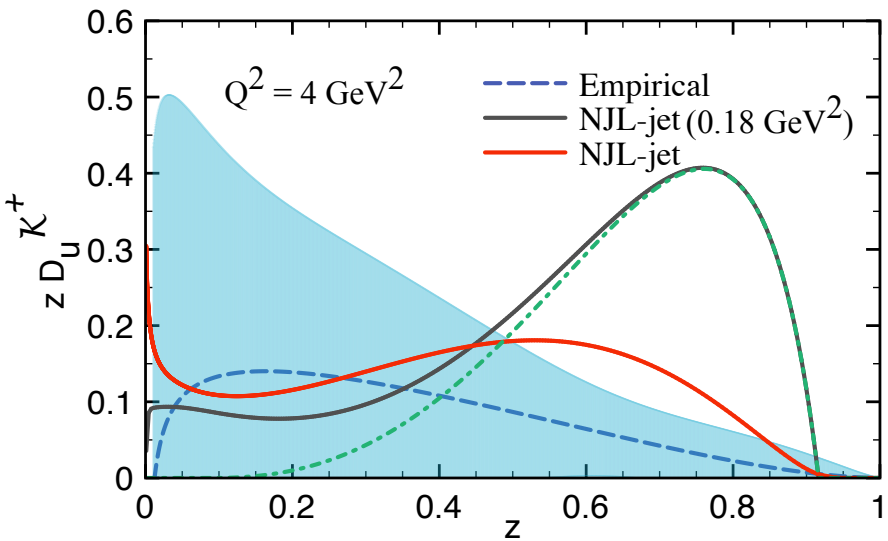


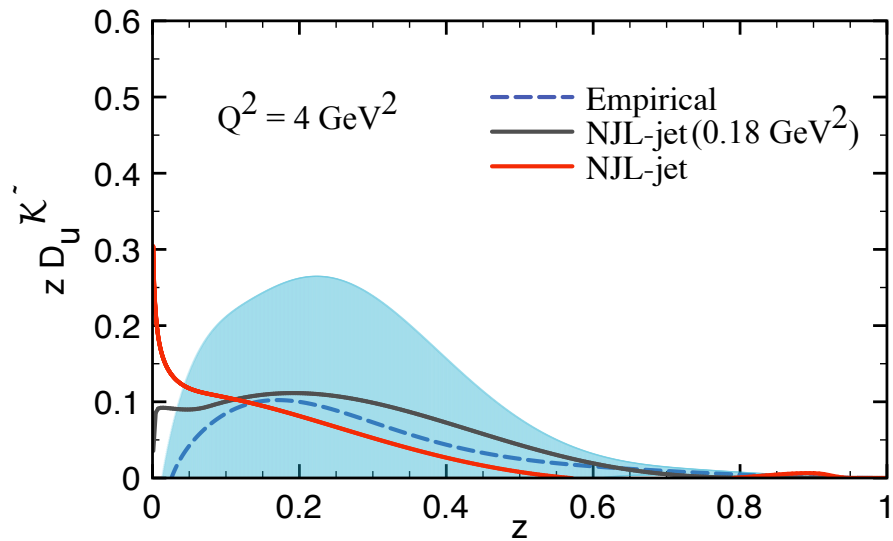






Results for Kaon





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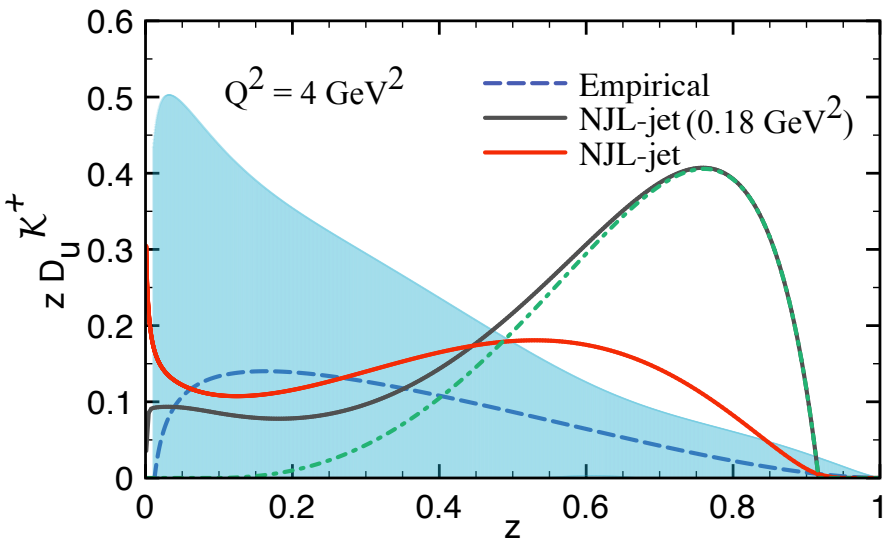
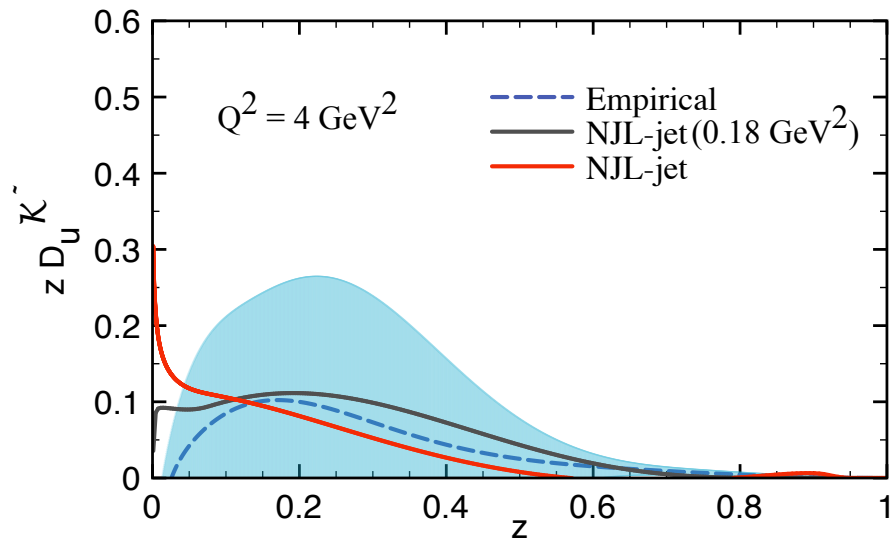
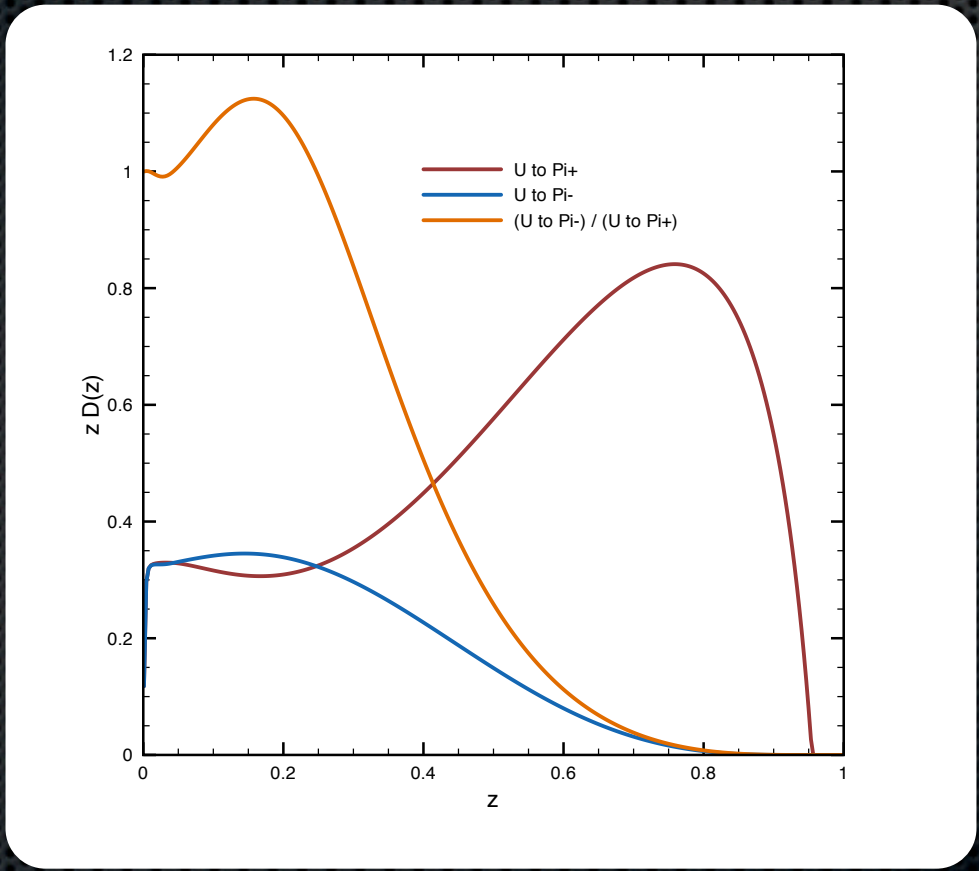




Momentum and Isospin Sum Rules Satisfied!

Unfavored to Favored Ratio

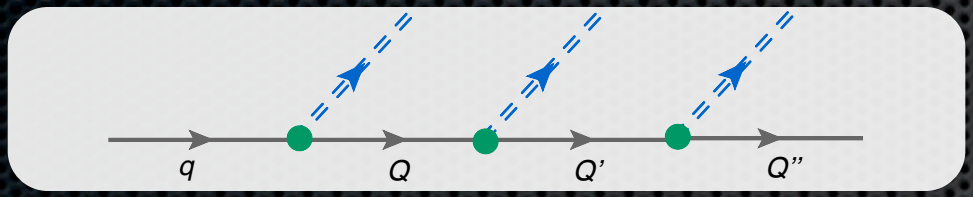


Monte-Carlo (MC) Simulations

- Simulate decay chains to extract probabilities
- Allows for inclusion of $p_\perp$
- Numerically trivially parallelizeable (MPI, GPGPU)

Fragmentations from MC

- Assume Cascade process:



- Randomly sample z from input splittings.
- Evolve to sufficiently large number of decay links.
- Repeat for decay chains with the same initial quark.

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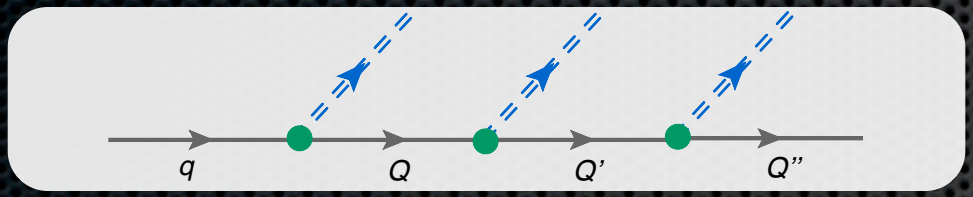

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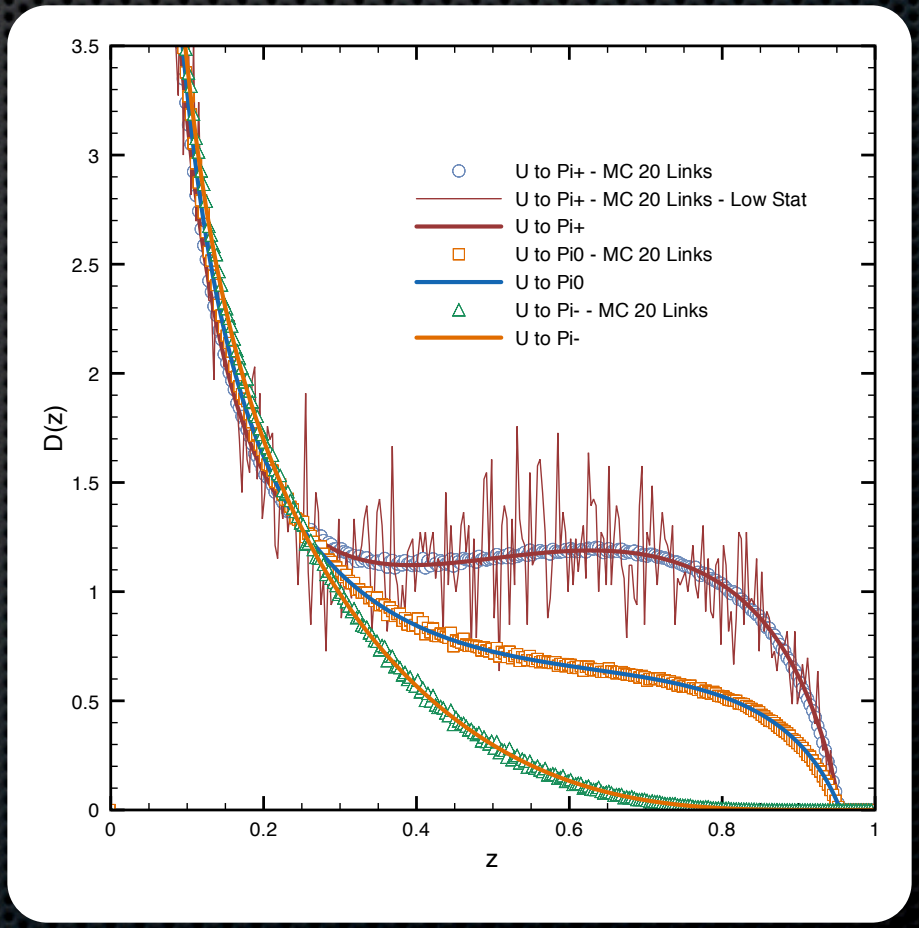
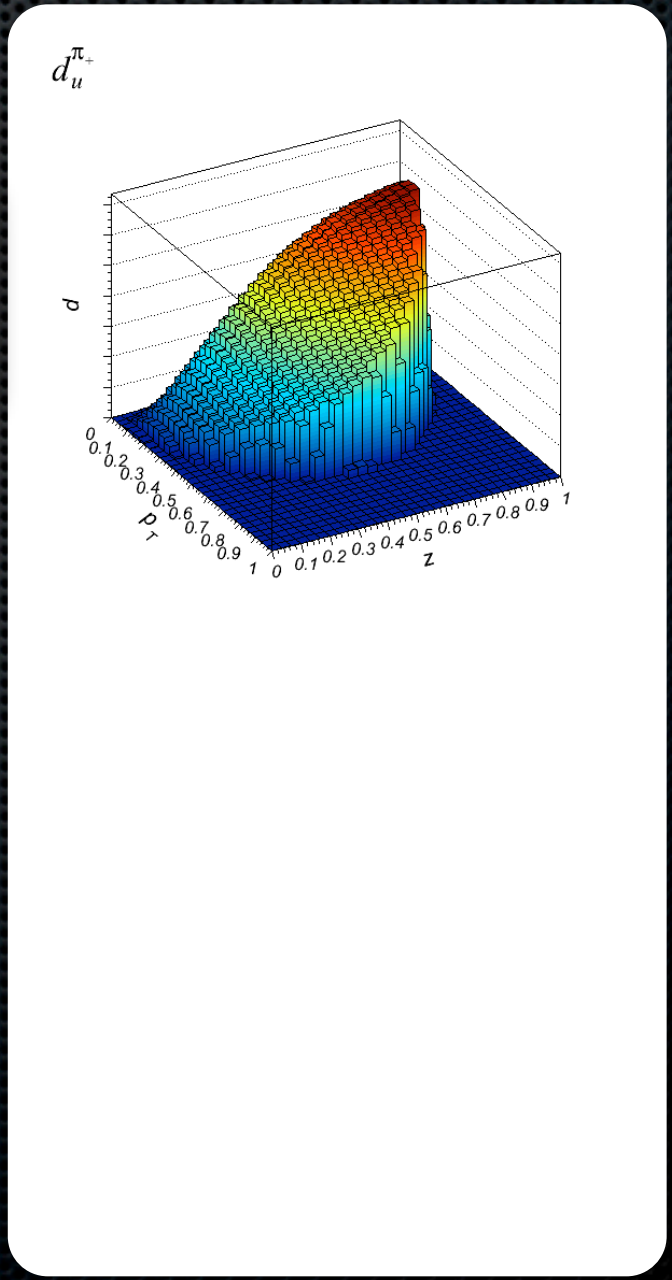


Including Transverse Momenta

- $p_\perp$ -dependent splittings: $d_q^m(z, p_\perp)$

$$\frac{C_I}{2} g_{mqq}^2 z \frac{p_\perp^2 + ((z-1)M_1 + M_2)^2}{(p_\perp^2 + z(z-1)M_1^2 + zM_2^2 + (1-z)m_m^2)^2}.$$



- Conserve transverse momenta at each link.

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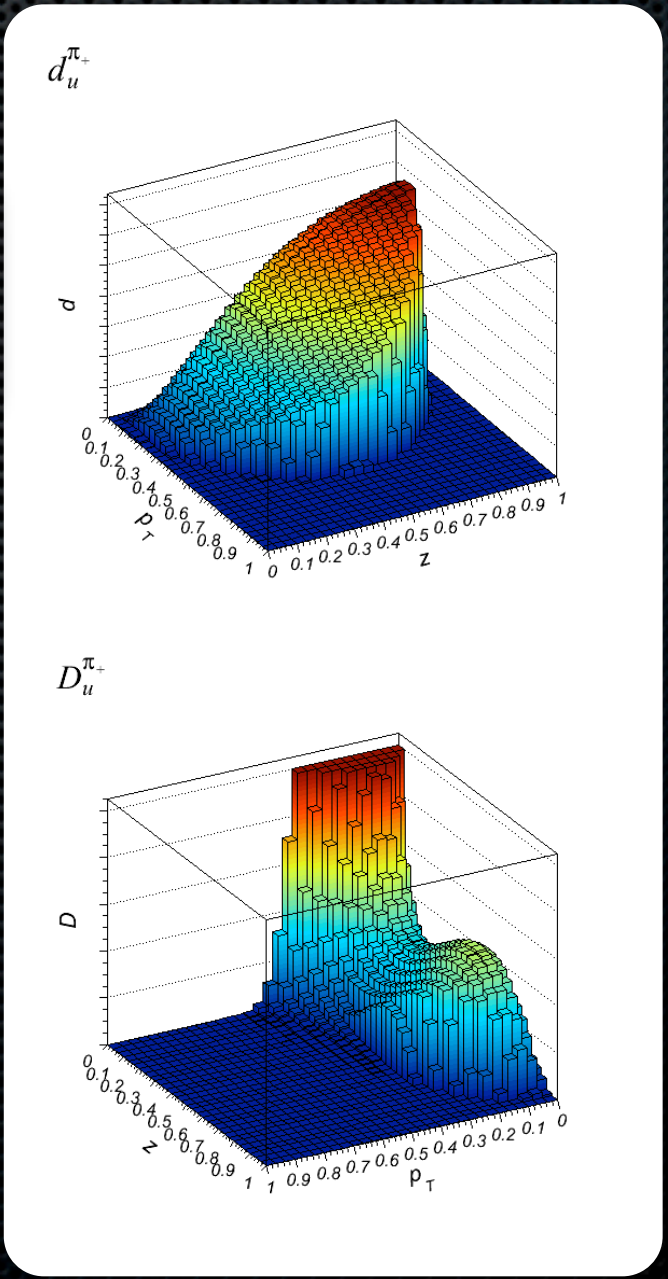
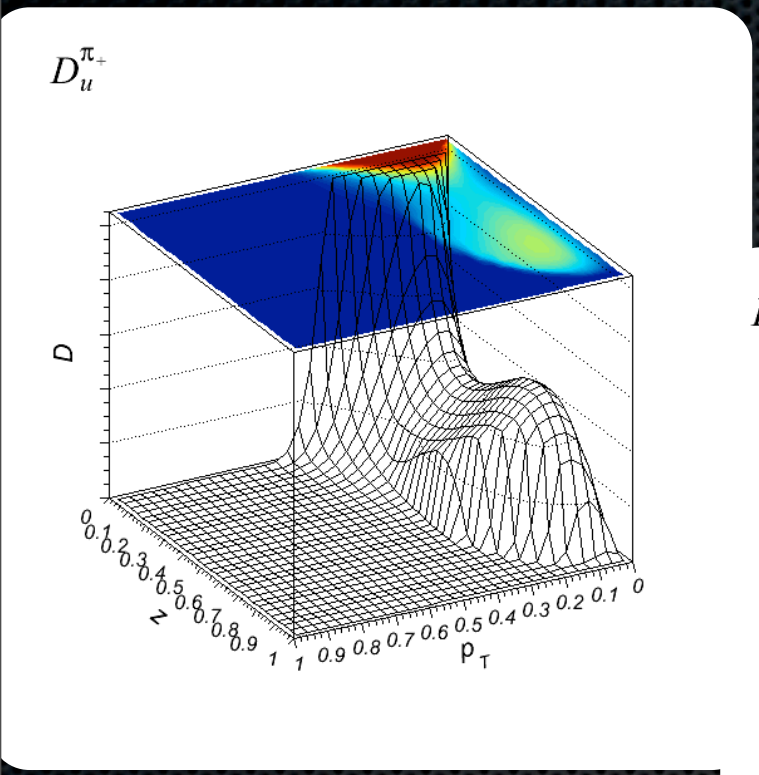
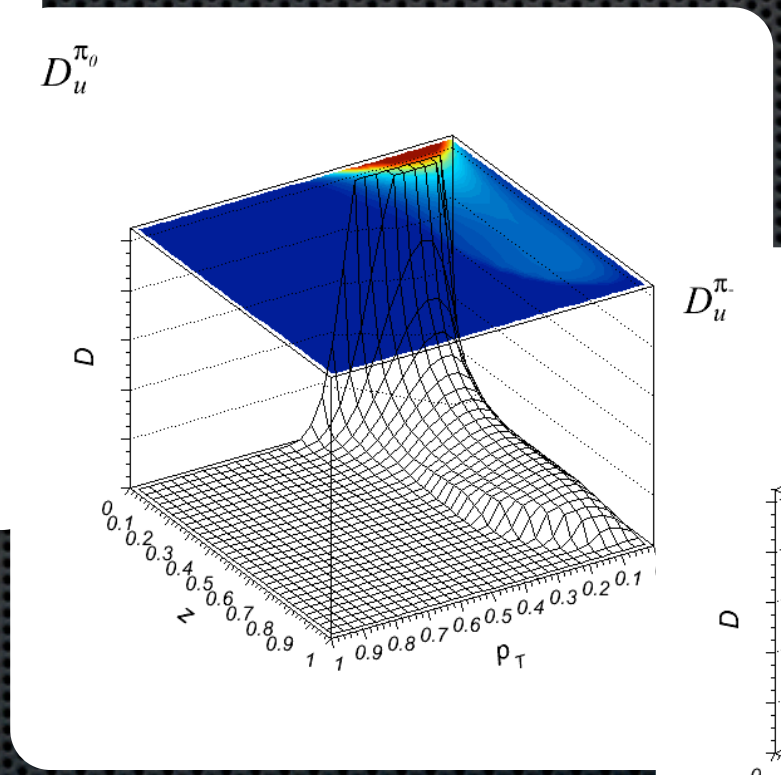
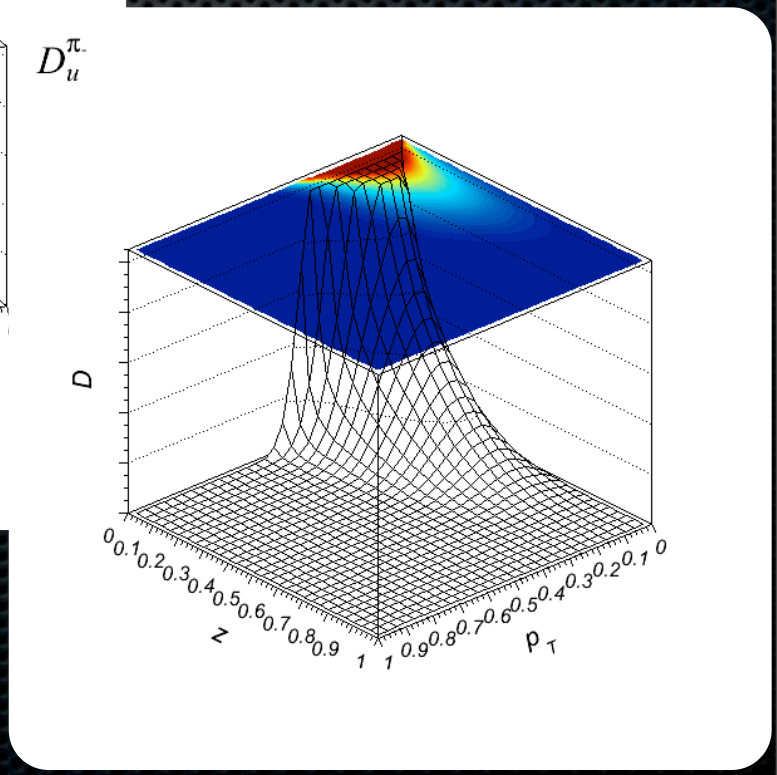


More Results with $p_\perp$







Outlook

- NJL-jet model: coupled channel cascade description of fragmentation function.
- Preliminary results: qualitative agreement with empirical parametrizations of experimental data.
- Improvements: Include vector mesons and ~~baryons~~ in fragmentation process, NLO DGLAP evolution.
- Improvements in MC.
- Polarized Fragmentation Functions